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Critical Comments on the Discussion about Tachyonic Causal Paradoxes and on the Concept of Superluminal Reference Frame

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The discussions of the tachyonic causal paradoxes and the concept of superluminal reference frame are criticized. The essence of the construction of the known paradoxes is revealed. Some possibilities of eliminating these paradoxes without contradicting the theory of relativity, are discussed. The tachyonic causal loop in an arbitrarily dimensional flat space-time is formally defined. The logical relations between assumptions on existence (or nonexistence) of the tachyonic causal loops and of inertial reference frames preferred in the tachyon kinematics are given. Such frames are not preferred in relation to bradyons and luxons, and maybe are not preferred in the dynamics of the tachyons. The theorem is proved which shows that the discussion on the tachyonic causal loops concerns also the preferred frames. The operational definitions of spacelike, timelike, and null vectors are given. It is shown that superluminal transformations and reference frames do not exist inside the theory of relativity. It is also shown that the so-called superluminal Lorentz transformations are not in fact transformations but mappings. It is concluded that the existence of tachyonic phenomena is not contradictory to the theory of relativity, while the concept of usual superluminal reference frame is contradictory to that theory.

1. INTRODUCTION

After the publication of the work by Bilaniuk et al. (1962) an avalanche of contributions on tachyons followed. Large parts of the bibliography have been given by Feldman (1974) (up to 1972), Recami and Mignani (1974) (up to 1974), and in a less detailed way in a survey by Recami (1978) (up to 1977). Only a small part of those works deals with quantum physics or general relativity. The large majority is concerned with special relativity. In

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this latter group two problems decidedly dominate, viz. that of tachyonic causal paradoxes and the one of superluminal frames and transformations. The present paper deals with those two subjects.

The trouble related to those problems has been the source of a great diversity of opinions and of fierce disputes. Besides, what is worse, the literature is infested with a large number of childish works, with long sequences of publications repeating the same ideas and even with intrinsically contradictory papers. In that chaos both interesting and brilliant ideas (though sometimes shocking from the contemporary point of view) as well as voices of reason get lost. The critical analysis of the situation made in the present paper will perhaps assist the interested reader in finding his way through that jungle.

Of course, somebody who would like tachyons to exist but at the same time thought that their existence leads to paradoxes [note that causal paradoxes are also considered in tachyonless physics (Wheeler and Feynman, 1949)] or to other inconveniences in terms of the theory of relativity, should create a new theory which would describe the phenomena occurring in the world of bradyons and luxons equally well, but which would eliminate the mentioned difficulties. Unfortunately we do not have such a theory, so in our further considerations we shall keep to the theory of relativity, limiting ourselves to its most basic level, i.e., to the *kinematic level*.

In Section 2, apart from general comments on the concepts of cause and effect, the essence of construction of the known tachyonic causal paradoxes is revealed and some known attempts to eliminate those paradoxes are criticized. Three ways of eliminating those paradoxes, noncontradictory to the theory of relativity, are discussed.

In Section 3 the logical relationships are presented occurring between the assumptions on the existence of causal loops and of preferred frames. To avoid gaps and doubts the convention of a short formal-like argumentation has been adopted in Section 3.1 which not only provides accuracy but also allows us to reveal the probably minimal set of assumptions. The elementary logical and mathematical terms used in Section 3.1 should not cause any trouble to physicists; however, if so the reader can pass directly to Section 3.2, where the principal conclusions drawn from Section 3.1 are discussed in conventional speech.

In Section 4, apart from the discussion on the concept of reference frame and on the differences between the concepts of transformation and mapping, the results of adding the concept of superluminal reference frame to the theory of relativity are analyzed from the operational point of view. Works supporting the concepts of superluminal reference frame and multidimensional time are also criticized in that section.

2. THE TACHYONIC CAUSAL PARADOXES: CAN THEY BE ELIMINATED OR NOT?

2.1. Some General Comments. It should be strongly emphasized that the existence of causal loops in the theory of relativity is not an intrinsic contradiction of that theory. For example, there exist curved space-times (with metrics which are solutions of the Einstein equations) in which closed timelike curves occur (Gödel, 1949; Carter, 1968). The concepts of cause, effect, and spontaneousness are not immanent to the theory of relativity. At the geometrical level only the concept of *event* (space-time point) and the concepts of sets of events, e.g., curves, surfaces, etc., are immanent to the theory. Space-time is the frozen field of events. If we have two events connected by a continuous, nowhere spacelike curve, then we can decide for the given space-time which of them is the cause and which the effect only after introducing a local coordinate system (chart) determining the time direction on that curve or in its neighborhood. Thus, in terms of the theory of relativity the estimates as to what is the cause and what the effect are taken arbitrarily from outside that theory, since we can always introduce another chart which will reverse that time direction. This causality is based on the concept of time direction. The concepts of cause and effect, however, can be introduced without time correlations (Benford et al., 1970), for instance, by observing phenomena in certain systems of objects, as shown very suggestively by Newton (1967) and discussed in greater detail by Csonka (1970). In Newton (1967) it is also shown how some phenomena, which may be regarded as spontaneous (Feinberg, 1967), can be treated as causally related. Thus we see that the concepts of cause and effect are not well defined in physics, being of psychological nature, probably secondary with respect to our intuitions related to natural science empiricism. However, our seemingly most obvious intuitions connected with that empiricism have already sustained two crushing defeats on the part of the theory of relativity and quantum physics. It seems that the trouble with the principle of causality may lead to a new confusion in the world of concepts similar to those just mentioned and to that experienced by the ancient Egyptians at the Euphrates riverside (Csonka, 1969). The only solid ground we have is *logical consistency*.

Would there be any known tachyonic causal paradoxes based on so weak a grounds as the concepts of cause and effect? No.

2.2. The *DEL* System and Criticism of Certain Attempts of Eliminating the Paradoxes. The authors of the known paradoxes use the concepts of cause and effect, free will of the experimenters, explosions in the laboratories, etc. only by way of illustration. Eventually they construct in their mental experiments the *DEL* system, which is a combination of the *DE* arrangement with the tachyonic causal loop *L* (for definition of such a loop see *D5* in Section 3.1).

The mental arrangement *DE* consists of a detector of tachyons *D* and an element *E* (emitter or transmitting detector of tachyons or else a diaphragm—no diaphragm setup or anything else), which acts at an event λ_E and is switched off at an event λ_{SO} if *D* detects a tachyon at an event λ_D. More precisely, the events λ_D and λ_{SO} are connected by a nowhere spacelike continuous curve K_1, and the events λ_{SO} and λ_E are connected by a timelike continuous curve K_2 (K_2 is a segment of the world line of *E*), and a past–future orientation is from λ_D to λ_{SO} and from λ_{SO} to λ_E. The events λ_D and λ_{SO} may coincide [then K_1 would reduce to the point $\lambda_D(=\lambda_{SO})$], while λ_{SO} and λ_E do not coincide, i.e., $\lambda_D \leqslant \lambda_{SO}$ along K_1 and $\lambda_{SO} < \lambda_E$ along K_2.

The *DEL* system is brought into being by the postulate that $K_1 \cup K_2 \subset L$. In the literature many mental realizations of that system are described in texts both supporting and criticizing the paradoxes. The existence of the *DEL* system implies the existence of a logical inconsistency by yielding the following conjunction of contradictory sentences: emission (passage) of tachyon at λ_E *and* no emission (no passage) of tachyon at λ_E (or conjunction: detection of tachyon at λ_D *and* no detection of tachyon at λ_D). Various ways of generating that conjunction by the *DEL* system are given in the literature in conventional language; a lucid example using the formal notation is given by Rolnick (1969, Section II).

Since *DEL* yields a logical contradiction, the known tachyonic causal paradoxes can be eliminated only by disintegrating the *DEL* system. Any attempts involving various additional speculations which leave the *DEL* system intact are bound to fail. Thus neither does the argumentation involving the boundary conditions given in Csonka (1970, Section 3(ii)) eliminate the simple Tolman paradox,[1] nor does the explanation given by Root and Trefil (1970) and Trefil in Recami (1978) eliminate the generalized Tolman paradox[2]; also looking at the loop from only one reference frame (Parmentola and Yee, 1971) does not eliminate the Pirani paradox (Pirani, 1970), although some authors believe that the quoted paradoxes have been eliminated in those ways.

[1]The Tolman paradox (Tolman, 1917) was repeatedly discovered by many authors; the relevant references are given, e.g., in Recami and Mignani (1974).

[2]This paradox was given by Bohm (1965) and in a more elegant form by DeWitt (Bilaniuk et al., 1969). Recently it has been repeated by Maund (1979, Section 4) and by Basano (1980, Section 3), the latter having introduced a certain stochastic process instead of the switch-off at the event λ_{SO} (in our terminology). In the following we shall refer to that paradox as the DeWitt one.

Some authors believe persistently that the famous reinterpretation principle is a remedy for the tachyonic causal paradoxes despite many works revealing the falseness of such a belief.[3] Eventually that principle is obtained by a speculation which leaves the *DEL* system intact (note that the reinterpretation principle incorporates in a significant manner the dynamic concepts, whereas the *DEL* system is constructed at a more fundamental level of kinematic concepts). The reinterpretation principle does not eliminate even the simple Tolman paradox, since an emission of the negative energy is a detectable event.

Our approach, and that of almost all authors, to the problem of tachyonic causal paradoxes has been based on the standard idealization, where we speak of phenomena at points or on single curves in space-time. The *DEL* system is presented in this spirit where the *DE* arrangement operates discretely in the yes–no mode, and the *L* loop is a curve. It seems that such an idealization is quite obvious, at least in the macroscale (i.e., in the macroregion of a space-time; see Section 2.3).

Nevertheless, Schulman (1971) assumed a different standpoint. He adopted in the attempt to eliminate the tachyonic causal paradox a method which was earlier applied in the contest against the nontachyonic causal paradox (Wheeler and Feynman, 1949). That method consists in a rigorous keeping to the model of continuity in nature. Though in Schulman's reasoning a significant role is played by the intensity of a tachyonic signal [more precisely, the continuous change of intensity in the threshold region; see Schulman (1971, p. 482, column 1)], what formally exceeds our kinematic level of considerations, but the continuous distribution of that intensity can be presented in a simple way in kinematic terms. Eventually, we can replace the world line of the tachyonic signal by a congruence of curves. Every curve corresponds then to a certain value of intensity, and those values continuously change across that congruence. In the spirit of the model of continuity in nature we can modify the *DEL* system by assuming that L is a congruence of closed curves and by replacing the points λ_{D}, λ_{SO}, and λ_{E} by the segments Λ_{D}, Λ_{SO}, and Λ_{E} of the congruence L. However, if we operate in the macroscale then for the given maximum width of L (orthogonality is an invariant) and the given lengths (along L) of the segments Λ_{D}, Λ_{SO}, and Λ_{E} we can always take a length of L so large (e.g., we can always make the fragments K_1 or K_2 sufficiently long by a suitable

[3]The reinterpretation principle with respect to tachyons has been presented in Bilaniuk et al. (1962) and favorably discussed in greater detail in Recami and Mignani (1974) and by Recami and Ziino (1976). Recently, e.g., Maund (1979) and Basano (1977, 1980) criticized that principle as a remedy against those paradoxes.

programming of the *DE* arrangement; here we are speaking of width and length in the Minkowski space), that practically the congruence L will be a curve, and Λ_D, Λ_{SO}, and Λ_E will be points. In this way we return to the standard *DEL* system and the paradox remains. Let us note that the real trick of Schulman consists in the implicit postulate that the segments Λ_{SO} (according to him the region between "on" and "off") and Λ_E (according to him the time A) overlap ["..., the switch will be in some intermediate position at the time A" (Schulman, 1971, p. 482, column 1)], and not in the existence of a meeting point. If Λ_{SO} and Λ_E overlap, then the afore-described return to the standard *DEL* system gives in practice $\lambda_{SO} = \lambda_E$. This means that the Schulman system is not the *DEL* system since in the latter we have $\lambda_{SO} < \lambda_E$ (along K_2). Indeed, if λ_{SO} is not earlier than λ_E (i.e., $\lambda_{SO} \geqslant \lambda_E$ along $K_1 \cup K_2$), then we have no causal paradoxes. Thus the elimination of the paradox in Schulman (1971) has consisted in using such a system that does not yield the paradox by definition, and not in using the model of continuity in nature. However, we can always assume the existence of *DE* arrangement such that Λ_{SO} and Λ_E are disjoint and Λ_{SO} is earlier than Λ_E (in the macroscale; as regards microscale, see Section 2.3). Thus the key postulate of Schulman (1971) (saying that the intermediate position of the switch determines the intensity of the tachyonic signal) collapses and the paradox remains.

2.3. Possibilities of Disintegrating the *DEL* System. The empirical realizations of the nontachyonic *DE* arrangement are encountered in every-day practice, for instance, when switching light off in the room. If tachyons were to interact with ordinary matter, then the empirical realization of the tachyonic *DE* arrangement would be equally obvious in the macroscale (having adequate technical possibilities). Let us assume that tachyons exist and interact with ordinary matter.[4] Since the *DEL* system yields a logical contradiction it cannot be empirically realized. Would the assumption of tachyons existing and interacting with ordinary matter be therefore contradictory to the theory of relativity? No. It suffices to show that it is theoretically possible to decompose the *DEL* system consistently with the theory of relativity. Such possibilities are discussed below.

Let us introduce the following definitions.

The region of space-time is a macroregion if and only if a *DE* arrangement may exist inside that region.

A region that is not a macroregion is called a microregion.

[4]We reject the opinion presented in the last phrase of Rolnick (1969) stating that if tachyons did exist they could not interact with ordinary matter, since then it would make no sense to speak of tachyons in physics, which, as a matter of fact, has been stated in Rolnick (1969, Section II).

Hence, inside a microregion by definition no arrangement *DE* may exist. The existence of microregions is not excluded. For instance, in accordance with our contemporary opinions it seems natural to suppose that in very small regions, where the laws of quantum physics have to be obeyed, the existence of an arrangement with a programmed control is impossible, or to suppose that in such regions idealization cannot be introduced by using pointlike events but such a continuous model should be applied in which the segments Λ_D, Λ_{SO}, and Λ_E would have to overlap (see Section 2.2). It seems that such suppositions (or maybe even conclusions) would have their source rather in quantum theories than classic relativity[5]; however, they would have to be of course consistent with the latter.

Thus the noncontradictory to the theory of relativity disintegration of the *DEL* system reduces to the following possibilities.

(α) The *L* loops exist inside microregions only. Then the system *DEL* and the known paradoxes do not exist.[6] Eventually, it might occur that we have discovered phenomena in which tachyonic causal loops appear (whose existence is not an intrinsic contradiction of the theory of relativity; cf. Section 2.1) and that would simply be an empirical fact (cf. Newton, 1967). However, a consistent theory (contained in the theory of relativity or in its extension) of those phenomena would impose constraints rendering impossible the expansion of the loop *L* beyond the microregion. For instance, it might occur that tachyonic phenomena have only existed in very small (Barashenkov, 1976), properly isolated regions of space and time. Creation of examples is a question of imagination; for instance, we can assume the existence of constraints due to the invariant uncertainty principle or to other similar but still unknown ones, etc. Meanwhile, in the macroscale tachyons would not be flying and everything would remain as it was.

(β) The *L* loops exist in macroregions, but no loop *L* may be combined with any *DE* arrangement to yield a *DEL* system. That possibility does not seem to be realistic but it might possibly be justified consistently with the relativity either for singular regions of a curved space-time or for a space-time curved so strongly that its local quasiflat regions would be microregions. These are, however, poorly convincing examples (e.g., the existence of overlaps of those quasiflat regions) and they are given here only with the

[5]Somebody might think that the use of the conditional form here is an excessive prudence, and that classical relativity can be *a priori* eliminated from such a game. That is not obvious, however. For instance, can one assume the existence of a nontachyonic *DE* arrangement inside such a region of space-time where closed timelike curves occur?

[6]Maybe it is also possible to construct tachyonic causal paradoxes in the absence of the *DEL* system, but I do not know that; here, however, only the known causal paradoxes are attacked in which the *DEL* system eventually operates.

hope that they may inspire somebody with an interesting idea. On the other hand that possibility seems to be nonexcluded even with respect to the flat space-time. For instance, the simple Tolman paradox is eliminated by the physically very suggestive assumption that a tachyon cannot be *directly* registered by a detector being at rest in a reference frame where the tachyon "moves backwards in time" to use the awful jargon. Such an idea can be found in the paper by Pavšič et al. (1976). Under that assumption the L loop present in the simple Tolman paradox cannot be connected with DE arrangement occurring in that paradox. The same L can be used in the DeWitt paradox, which, however, includes a different DE arrangement with which that L can already be connected to yield the DEL system. Therefore that assumption does not eliminate the DeWitt (nor the Pirani) paradox. Perhaps somebody will conceive a similar, more general idea? The considered possibility though not excluded seems to be, however, as we have already said, little probable.

(γ) The L loops do not exist. That possibility is related to the existence of an inertial reference frame preferred with respect to tachyons, and it is accurately analyzed in Section 3.

It should be emphasized that the above possibilities cannot be rejected *a priori* as impossible in view of logical contradictions, at least the possibilities (α) and (γ). The settlement lies in experiment.

3. THE TACHYONIC ETHER AND THE TACHYONIC CAUSAL LOOPS

This section concerns the kinematics of the pointlike tachyons having straight world lines (i.e., having constant velocity) in the flat space-time M, which is infinite in every direction, homogeneous, spatially isotropic, and has a determined past–future orientation. The M can have arbitrarily chosen number of spacelike dimensions and one timelike dimension only (usual Minkowski's space).

3.1. Formal-Like Considerations. The capital letters C, D, L, and P followed by numbers mean, respectively, a corollary, a definition, a lemma, and a postulate. The term *if and only if* (equivalence) we abbreviate *iff*.

The Meaning of Symbols.

$\lambda_1, \lambda_2, \ldots$ are points of M (events; thus symbols $\lambda_1, \lambda_2, \ldots$ and $\lambda_1 \in M, \lambda_2 \in M, \ldots$ are synonyms, respectively);

$A, B, \ldots$ are segments of nowhere spacelike continuous curves;

K is a continuous curve;

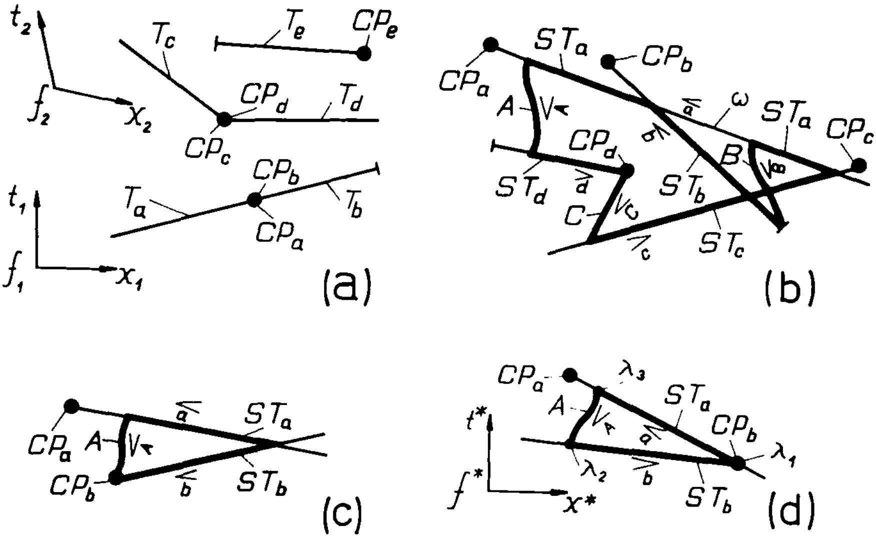

Fig. 1. The illustrations are for two-dimensional M. (a) T_a, T_c, and T_d are the half-infinite lines, while T_b and T_e are the segments (short lines may represent, e.g., opaque obstacles); tachyons a, b, and c, d, have common creation points, respectively; $f_1 \in \mathbb{F}_{-a} \cap \mathbb{F}_{+b} \cap \mathbb{F}_{+c} \cap \mathbb{F}_{+d} \cap \mathbb{F}_{+e}$, $f_2 \in \mathbb{F}_{-a} \cap \mathbb{F}_{+b} \cap \mathbb{F}_{+c} \cap \mathbb{F}_{+d} \cap \mathbb{F}_{-e}$, f_1 is the critical frame for d; if $f_1 \in \mathbb{F}_p$, then a cannot exist; if $f_2 \in \mathbb{F}_p$, then neither a nor e can exist; if $\mathbb{F}_p \neq \emptyset$, then either a or d cannot exist. (b) An example of tachyonic causal loop (the thick closed curve); if one wishes, one can formally exclude from that loop the ST_a which is identical to ω (as it has been done here), as thanks to the presence of ST_b and B the curve remains closed and suitably oriented (see $D5$). (c) The thick closed curve is not a tachyonic causal loop because of the direction of $_b <$. (d) Illustration to the proof of the theorem; the thick closed curve is the simplest tachyonic causal loop.

$a, b, \ldots$ are tachyons;
$T_a, T_b, \ldots$ are world lines of $a, b, \ldots$;
$ST_a, ST_b, \ldots$ are segments of $T_a, T_b, \ldots$;
$CP_a, CP_b, \ldots$ are creation points (events) of $a, b, \ldots$;
f, f_1, f_2, f^* are inertial reference frames in M (thus they are connected by the time-irreversible Lorentz transformations)[7];
$\mathbb{F}$ is a set of all f's (thus symbols f, $f_1, \ldots$ and $f \in \mathbb{F}$, $f_1 \in \mathbb{F}, \ldots$ are synonyms, respectively);
$\mathbb{F}_{+a}, \mathbb{F}_{-a}, \mathbb{F}_{+b}, \mathbb{F}_{-b}, \ldots$ are subsets of $\mathbb{F}$ (see $D3$ and $D4$, below);

[7]We assume that two f's are different iff they move relatively, thus transformations of only spatial coordinates do not change any f. For definition of f see Section 4.2. We use here only the usual subluminal reference frames and transformations rejecting the superluminal ones for reasons which will be given in Section 4.4.

$\mathbb{F}_p$ is a set of all f's which are preferred with respect to tachyons ($\mathbb{F}_p \subset \mathbb{F}$; see *D6*);

$\mathbb{F}^*$ is a set defined by *D7* ($\mathbb{F}^* \subset \mathbb{F}$);

$\mathfrak{L}$ is a set of all tachyonic causal loops in M;

$\mathbb{O}, \mathbb{\Theta}$ are empty sets;

$\lambda_1{}^f\!< \lambda_2$ means that λ_1 precedes λ_2 in time in f;

${}_A\!<$ denotes the time sequence of events along A (see *D1*);

${}_a\!<$ denotes the sequence of events along T_a in relation to CP_a (see *D2*).

P1. For every a, T_a is a semi-infinite straight spacelike line beginning at CP_a or a straight spacelike segment of which one and only beginning point is CP_a (see Figure 1a).

D1. $\lambda_1{}_A\!< \lambda_2$ iff $\lambda_1 \in A$ and $\lambda_2 \in A$ and there exists f such that $\lambda_1{}^f\!< \lambda_2$.

C1. If there exists A such that $\lambda_1{}_A\!< \lambda_2$, then for every f, $\lambda_1{}^f\!< \lambda_2$.

D2. $\lambda_1{}_a\!< \lambda_2$ iff $\lambda_1 \in T_a$ and $\lambda_2 \in T_a$ and segment $[CP_a, \lambda_1]$ is shorter than segment $[CP_a, \lambda_2]$.

C2. The relations ${}_A\!<$ and ${}_a\!<$ are invariants of the time-irreversible Lorentz transformations.

D3. $f \in \mathbb{F}_{+a}$ iff: for all λ_1 and λ_2, if $\lambda_1{}_a\!< \lambda_2$, then $\lambda_1{}^f\!< \lambda_2$; or f is a critical frame for a (i.e., a has an infinite speed in f).

D4. $\mathbb{F}_{-a} = \mathbb{F} - \mathbb{F}_{+a}$.

Thus, if $f \in \mathbb{F}_{-a}$, then some people unhappily say that a "moves backwards in time" in f.

L1. For every a, $\mathbb{F}_{+a} \neq \mathbb{O}$ and $\mathbb{F}_{-a} \neq \mathbb{O}$.

The proof results from *D3*, *D4*, and the transformational properties of spacelike lines with respect to relation ${}^f\!<$.

The concept of a specific point named the creation point of the tachyon is crucial in our considerations. At the level of special relativity this is a primitive concept (introduced by *P1*) but in general relativity the existence of such point can be regarded as a *conclusion* (Kowalczyński, 1979). Owing to the existence of CP_a we can introduce the relation ${}_a\!<$ by *D2* and then divide the set $\mathbb{F}$ on subsets $\mathbb{F}_{+a}$ and $\mathbb{F}_{-a}$ by *D3* and *D4*. Thus every T_a is distinguished from the usual geometrical spacelike line or a segment as an invariantly directional (by ${}_a\!<$; see *C2*) world line. CP_a has its own sense (of being a creation point) in every $f \in \mathbb{F}_{+a}$, and has a kinematic sense of an annihilation point of the tachyon a in every $f \in \mathbb{F}_{-a}$ [except of the critical frame belonging to $\mathbb{F}_{+a}$ by *D3*, where CP_a has a sense of a creation–annihilation point; see Figure 1a and Kowalczyński (1979)].[8]

D5. $K \in \mathfrak{L}$ iff K is closed and there exist $a, b, \ldots,$ and there exist $A, B, \ldots,$ such that: K consists of ST_a, $ST_b, \ldots,$ and $A, B, \ldots,$ and only of them; and ${}_a<$, ${}_b<, \ldots,$ and ${}_A<$, ${}_B<, \ldots,$ have the same direction along K (see Figure 1b, c).

Note that the concept of f does not appear in *D5*. See also *C2*.

C3. If $K \in \mathfrak{L}$, then for every f there exists a such that $ST_a \subset K$ and $f \in \mathbb{F}_{-a}$.

D6. $f \in \mathbb{F}_p$ iff for every a, $f \in \mathbb{F}_{+a}$.

L2. If $\mathbb{F}_p \neq \emptyset$, then $\mathfrak{L} = \emptyset$.

The proof results from *D6* and *C3*.

Up to now we have not made any assumptions about a number of tachyons in M, e.g., there could exist only one tachyon, and we have not used any modal term in our simple *C*'s, *D*'s, *L*'s, and *P1*. Now we introduce the modal term *may* (possibility) in order to make the considerations that will follow, shorter and much clearer.

D7. $f \in \mathbb{F}^*$ iff there exists an emitter of tachyons such that it may be at rest in f, and if it is at rest in f, then it may emit a tachyon in every arbitrarily chosen spatial direction in M with an arbitrary speed (obviously, greater than the speed of light) as measured in f.

A sense of the term *to emit* implies the following: if a certain a is emitted by an emitter being at rest in f, then $f \in \mathbb{F}_{+a}$.

D1–*D6* are normal definitions expressed in mathematical terms of special relativity.[9] *D7* is also a normal [due to its structure (equivalence)] definition being at the same time an operational and real (in the object stylization) definition.

[8] The physical interpretation of the tachyonic motion by virtue of which a creation point of the tachyon preserves its physical sense in *all* f's is given in Kowalczyński (1979, Section 2). This interpretation implies that every T_a has the physical sense, in every f, of a faster-than-light directional (by ${}_a<$) signal from a physical source CP_a.

[9] Note that the names of the physical (hypothetical) objects $a, b, \ldots$ appear in our formal expressions in an insignificant way because the symbols $a, b, \ldots$ play the role of common indices only, which connect accordingly those mathematical terms. The names of the tachyons could be substituted by the names of their world lines, but such a procedure would make the expressions less clear. Note also that in fact f's are mathematical objects (see Section 4.2).

C4. If $f_1 \in \mathbb{F}^*$ and $f_1 \neq f_2$, then there exists a[10] such that $f_2 \in \mathbb{F}_{-a}$.

C5. If $\mathbb{F}^* \neq \mathbb{O}$, then for all f_1 and f_2 there exists a[10] such that $f_1 \in \mathbb{F}_{+a}$ and $f_2 \in \mathbb{F}_{+a}$.

L3. If $\mathbb{F}_p \neq \mathbb{O}$ and $\mathbb{F}^* \neq \mathbb{O}$, then $\mathbb{F}_p = \mathbb{F}^*$ and $\mathbb{F}_p$ possesses one element only.

The proof results from *D6*, *D7*, and *C4* by *reductio ad absurdum*.

The assumption that $\mathbb{F}_p \neq \mathbb{O}$ and $\mathbb{F}^* \neq \mathbb{O}$ (which needs not be made of course) means by virtue of *L3* that we allow for the existence of the *kinematic tachyonic ether*. Then the one and only element of $\mathbb{F}_p (= \mathbb{F}^*)$ is the f fixed in this ether. Note that *every* f is *not* a preferred frame in relation to bradyons and luxons ($\mathbb{F}_p \subset \mathbb{F}$), thus the assumption that the ether exists leaves the principle of relativity *intact* for all systems that are regarded as tachyonless.[11] Note also that the ether refers to the kinematics of tachyons and it is not obvious whether from the point of view of tachyon dynamics it is an ether at all, e.g., if we take the model of the tachyon like that presented in Kowalczyński (1979).[12] That problem we leave open.[13]

Our ether is spatially isotropic, as M is, and the speed of tachyons in it is unlimited. However, the condition $\mathbb{F}_p \neq \mathbb{O}$ is also not in contradiction to the concept of spatially isotropic ether with limited speed of tachyons in it, and to the concept of spatially unisotropic ether.[14] Thus the assumption that any such ether exists eliminates the tachyonic causal loops by virtue of *L2* and every such ether is not an ether for bradyons and luxons.

Let us assume for simplicity that the emitter of tachyons mentioned in *D7* is pointlike.

[10] More exactly we should use here the term *there may exist*, but if we assume that emission of such tachyon is (was, will be) executed, then we can use the term *there exists*.

[11] For the sake of simplicity we can also assume that those systems are considered to be free from virtual tachyons. See also Barrowes (1977, three last paragraphs of Section 4, and Sections 5 and 6).

[12] Especially cf. footnote 4 in Kowalczyński (1979) where the misprint "techyonic manifest" should read "tachyons manifest."

[13] The connections between such an ether, the cosmic background radiation, and the Mach principle, though very suggestive, for the moment can only be the subject of philosophical considerations. Perhaps, however, they will be determined in physics in the future.

[14] The concepts of preferred frames and of tachyon corridor introduced by Antippa and Everett (Marchildon et al., 1979) (for critical comments see Lee and Kalotas, 1977) are essentially different from our concepts of f's belonging to $\mathbb{F}_p$ and of the tachyonic ethers. Their idea involves the essential interchange of properties between space and time for tachyons, while we represent the orthodox stand. A few unfortunate ideas concerning preferred frames were published. The definition of the preferred frame given by Everett (1976) contains an intrinsic contradiction (cf. Section I and property C in Section II in Everett, 1976). Sudarshan in Recami (1978) describes the strange preferred frame which is a critical frame for all cosmic tachyons.

P2. If the emitter of tachyons mentioned in *D7* exists, then it may be placed in every arbitrarily chosen point of *M*.

Theorem. If *P2* and $\mathbb{F}^* \neq \mathbb{O}$, then; $\mathbb{F}_p \neq \mathbb{O}$ iff $\mathfrak{L} = \mathbb{O}$.

In one direction the proof follows immediately from *L2*. The proof in the opposite direction is done by *reductio ad absurdum*. In the latter case we would have that *P2* and $\mathbb{F}^* \neq \mathbb{O}$ and $\mathfrak{L} = \mathbb{O}$ and $\mathbb{F}_p = \mathbb{O}$. Let $f^* \in \mathbb{F}^*$. The f^* exists by condition $\mathbb{F}^* \neq \mathbb{O}$. By condition $\mathbb{F}_p = \mathbb{O}$ and by *D4* there exists *a* such that $f^* \in \mathbb{F}_{-a}$. Let $\lambda_1 \in T_a$ and $\lambda_1 \neq CP_a$ (see Figure 1d). Let the emitter being at rest in f^* ($f^* \in \mathbb{F}^*$) coincide with λ_1 (what can take place by *P2*). Then there exist *b* (see footnote 10), *A*, λ_2, and λ_3 such that $CP_b = \lambda_1$ and $\lambda_2 \in T_b \cap A$ and $\lambda_3 \in T_a \cap A$ and $\lambda_{1b} < \lambda_{2A} < \lambda_{3a} < \lambda_1$. Thus there exists the causal loop that contradicts the condition $\mathfrak{L} = \mathbb{O}$. Thus the theorem is proved.

The idea included in *P2* and in the condition $\mathbb{F}^* \neq \mathbb{O}$ (i.e., in the antecedent of the theorem) was essential for construction of all the known tachyonic causal paradoxes but probably was never formulated explicitly.

3.2. Discussion. In the preferred frames defined by *D6* no tachyons "move backwards in time" (by *D3*). The assumption that at least one such frame exists in *M* implies by *L2* that no tachyonic causal loop can exist in *M*, thus if one allows the existence of even one such loop, one rejects the possibility of such a frame (and hence tachyonic ether) existing in *M*. Thus the assumption that the preferred frame exists is way of eliminating the known tachyonic causal paradoxes[15] [cf. possibility (γ) in Section 2.3]. It should be strongly emphasized that this assumption is not contradictory to the theory of relativity. Our preferred frames are preferred only with respect to tachyons (and are defined at the kinematic level). As regards bradyons and luxons those frames are normal, nonpreferred inertial ones (see footnote 7); neither would tachyonic ether have any ether properties for all systems that are regarded as tachyonless (see footnote 11).

Were tachyons discovered and it occurred that the experiment analogous to the Michelson–Morley one could be realized, its result would decide either in favor of the tachyonic ether hypothesis or against it; however, today we cannot conclude as to that result. Nevertheless, the fact that the tachyonic ether hypothesis eliminates in exceptionally simple way the known

[15] The idea for elimination the tachyonic causal loops by the assumption that a preferred inertial frame exists was given by DeWitt (Bilaniuk et al., 1969), Barrowes (1977, Section 3.2), and by Lord and Shankara (1977, Section 3); but DeWitt's restriction concerning emission and absorption of tachyons seems to be too strong because of, e.g., *C5*.

causal paradoxes as well as the result obtained by Plebański[16] should be a warning for the enemies of the ether idea.

The implication opposite to *L2* cannot be proved[17] without suitable additional assumptions (let us call them operational). The domain of problems to which those operational assumptions pertain is shown, e.g., by the definiens of *D7* and by *P2*. In the constructions of the known causal paradoxes the operational assumptions were accepted as intuitively obvious (what is surely a weakness of those constructions from the methodological point of view; see Section 2.1). Eventually, the intuitive obviousness of the operational assumptions is very strong in those constructions. This results from the properties of the flat space-time, in which the paradoxes were constructed, i.e., homogeneity, infinity, spatial isotropy, and that the Lorentz transformations allow to find for every spacelike line in that space-time such a subluminal frame in which that line corresponds to the predetermined superluminal velocity. If we make some adequate operational assumptions, what was the case with both the authors of the known causal paradoxes and their opponents, then we can prove the implication opposite to *L2* (the theorem given in Section 3.1 can serve as example[18]). Then the postulate that the tachyonic causal loop exists (does not exist) is *equivalent* to the postulate that the tachyonic ether does not exist (does exist).

We see therefore that the discussion conducted hitherto on the tachyonic causal paradoxes related also to the embarrassing problem of existence of tachyonic ether (if somebody once had already allowed the assumption that tachyons do exist).

4. ON THE SUPERLUMINAL REFERENCE FRAME

The concept of a superluminal reference frame for the two-dimensional flat space-time has been introduced by several authors (Jones, 1963; Gilson, 1968; Mariwalla, 1969; Parker, 1969). That concept has been developed for

[16] Plebański (1970) has rigorously shown that the existence of a certain kind of ether for faster-than-light signals and of such signals themselves can be regarded as a *conclusion* from the nonlinear electrodynamics *included* in the theory of relativity. Such an ether is an electromagnetic field (thus it could be produced in a region of M) and is indispensable for the existence of such signals.

[17] For example, if there exist in M only two tachyons like a and d in Figure 1a, then neither a preferred frame nor a tachyonic causal loop exist in M.

[18] The equivalence in the consequent of that theorem can be also proved under operational assumptions other than those in the antecedent of that theorem. For example, the assumptions in that antecedent could be made weaker but then they would seem strange as regards the mentioned properties of M.

two and four dimensions by many authors and despite criticism[19] has become the basis for predicting sensational "physical" phenomena. Three principal trends were developed by: Recami et al. [innumerable number of works; for a bibliography see, e.g., Recami and Mignani (1974) and Recami (1978), and recently Caldirola et al. (1980)], Antippa and Everett (Marchildon et al., 1979), and Goldoni (1973, 1978).

4.1. The Difference between Mapping and Transformation. It seems that the basic methodological error consisting in that two essentially different concepts, i.e., transformation of the local coordinate system and mapping (function) of one manifold into or onto another manifold (called for short in the following transformation and mapping, respectively), are not distinguished is the source of misunderstanding. Eventually, let us consider the system of m equations

$$x^{\mu'} = Y^{\mu}(x^1,\dots,x^m) \tag{1}$$

determining the homeomorphism between two open sets X and X', which are sets of m-tuples, i.e., $(x^1,\dots,x^m)\in X$ and $(x^{1'},\dots,x^{m'})\in X'$. The Greek indices here and in the following are integers from 1 to m. The numbers x^{μ}, $x^{\mu'}$ (called coordinates in the following) as well as the functions Y^{μ} can in general be complex (see Appendix A). It is commonly known that equations (1) have in topology (and hence in differential geometry and thus in the theory of relativity) two different meanings.[20]

In the first meaning equations (1) determine in coordinate terms the mapping

$$\phi: M \to M' \tag{2}$$

where M and M' are m-dimensional topological manifolds which can *differ*. More precisely, the system of m functions Y^{μ} is the realization of the homeomorphism Y involving (U,φ) and (U',φ') charts, where $\phi = \varphi'^{-1}\circ Y\circ\varphi$, $U\subset M, U'\subset M'$, $\varphi(U)=X$, and $\varphi'(U')=X'$. It is not always possible to determine mapping (2) by equations (1), for instance, if ϕ is not a one-to-one mapping or, e.g., as in the known trick of transition from the Schwarzschild space-time to a completely different Levi-Civita space-time, where the mapping is realized by substituting the product of the real

[19] That concept was criticized among others by Naranan (1972), Yaccarini (1974), and by Lee and Kalotas (1977). Mignani and Recami (1974) defended themselves against the objections raised in Yaccarini (1974), however, their argumentation was faulty as we shall see further in this section and in Appendix B.

[20] This differentiation being obvious it is not usually distinctly underlined in textbooks of topology and differential geometry. This difference is strongly stressed by Gołab (1974, Section 3).

coordinate and number i for the real coordinate (Ehlers and Kundt, 1963) (cf. last paragraph in Appendix B). It is commonly known that the manifolds M and M' appearing in relation (2) may have different geometrical structures or be manifolds of different kind, as for instance in mapping (3) further in the text, even if ϕ is determined by equations (1). Specifically, if M and M' are space-times, there may exist a mapping (2) such that a congruence of timelike curves in M' is an image of a congruence of spacelike curves in M or vice versa.

In the second meaning equations (1) determine the transformation of local coordinates x^{μ} to local coordinates $x^{\mu'}$ on *one and only one* m-dimensional topological manifold M. More precisely, the system of m functions Y^{μ} determines the transition from chart (U, φ) to chart (U', φ') in a nonempty region $U \cap U'$, where U, $U' \subset M$, $\varphi(U) = X$, and $\varphi'(U') = X'$.[21] Transformation can always be expressed by equations (1) and it does not affect the structure of M (for additional information see Appendix B).

In other words, we can say that in the first meaning (mapping) the equations (1) determine a transition from a point of M to a point of M' (in the special case, when $M \cap M'$ is not an empty set and $U' \subset U$, we have a transition from a point of M to another point of M including the simplest case $x^{\mu'} = x^{\mu}$ for all μ, i.e., no transition occurs), while in the second meaning (transformation) the equations (1) do not change any point of M but merely alter the coordinates of the point (including the simplest case $x^{\mu'} = x^{\mu}$ for all μ, i.e., no change of the coordinates occurs) (Gołab, 1974, Section 3).

4.2. The Reference Frame in Mathematics and Physics. Here and in the following we shall concentrate only on a specific kind of topological manifold, i.e., the m-dimensional Riemannian space without singular points, also denoted by M [then the system of equations (1) is a diffeomorphism]. Thus for every point $\lambda \in M$ there exists its open neighborhood $U_{\lambda} \subset M$ such that U_{λ} can be treated as a flat space (then treatment consists in applying the standard method of neglecting the small quantities). The Riemannian space with respect to which the physical interpretation used in the theory of relativity has been applied (see Section 4.3) is called space-time. If M is a space-time, then U_{λ} can be treated as a Minkowski space (as a fragment or, after adequate one-to-one mapping, as a whole) and we can operate in U_{λ} in a simplified manner like in special relativity—for instance, make shifts or

[21] The definitions of such concepts as manifold, mapping (map, in terms used by some authors), chart, tangent vector space, basis, etc. can be found in modern textbooks of topology and differential geometry. They are given in a very condensed and lucid form in the book by Kramer et al. (1980), where standard drawings illustrating transformation and mapping (Figures 2.1 and 2.2, respectively, in Kramer et al., 1980) are also given.

determine the parallelism of lines. Besides, if M is a space-time, then—as it is commonly known—for every $\lambda \in M$ there exists $U'_\lambda \subset M$ such that $U_\lambda \subset U'_\lambda$ and in the region U'_λ we can introduce an infinite number of such local coordinate systems each of which is a Lorentz coordinate system at λ, i.e., $g_{\mu\nu}(\lambda) = \eta_{\mu\nu}$ and $g_{\mu\nu,\rho}(\lambda) = 0$, and can be treated as a Lorentz coordinate system in U_λ.

Let $\mathbb{V}_\lambda$ be the tangent vector space at arbitrarily chosen $\lambda \in M$. The set of m linearly independent elements of $\mathbb{V}_\lambda$, i.e., of m independent vectors fixed at λ, is called basis of $\mathbb{V}_\lambda$ (see footnote 21). The vectors of the basis are tangent at λ (by definition) to locally smooth curves contained in M. Thus, those vectors determine m independent directions in M at point λ. The basis of $\mathbb{V}_\lambda$ is called the coordinate basis of a local coordinate system in M (for short: coordinate basis at λ) if and only if the mentioned curves are coordinate curves of that coordinate system. The terms "coordinate basis (at λ)" and "reference frame (at λ)" are synonyms (cf.: Gołab, 1974, Sections 16 and 50; and Kramer et al., 1980). Then the point λ is called the origin of a reference frame. The vectors of the coordinate basis are called the axes of the reference frame.

As it is seen the concept of reference frame is defined by the concept of local coordinate system (a detailed discussion of the relationship between those concepts in mathematical terms is given in Gołab, 1974, Sections 16 and 50). Thus, changes of the reference frames are equivalent to changes of the coordinate systems on *one and only one* manifold, and hence those changes are realized via *transformations*.

In general, the reference frame is not orthogonal; however, in the theory of relativity the term *reference frame* is often understood more narrowly, i.e., as a frame of orthonormal vectors (Synge, 1964). That orthonormality is realized by the requirement that the basis vectors are tangent to the coordinate lines of the Lorentz coordinate system. Such a frame is then called *inertial reference frame* (Misner et al., 1973). The latter is denoted below by f, and the set of all f's whose origins are in U_λ is denoted by $\mathbb{F}_\lambda$. (In the case of special relativity, i.e., when the whole Minkowski space is used instead of U_λ, the subscript λ may be deleted and the symbols f and $f \in \mathbb{F}$ become synonyms as in Section 3.1).

The above definition of f is given in mathematical terms and is understood in relativistic physics as a mathematical model of f (cf. Misner et al., 1973). However, in the common understanding the term *inertial reference frame* is semantically connected with the concept of *rest* (immobility) and eventually in that meaning that term is used both in nonrelativistic and relativistic physics (cf. Synge, 1965, pp. 53, 113, and 117). That fact is emphasized by the use of the figurative expression "the observer being at rest in f." The possibility or rather the necessity of defining rest in f is an attribute of the concept of f. In other words, if rest cannot be defined, then there is no sense of speaking of f from the physical point of view.

The definition of rest in f, expressed in mathematical terms used in the theory of relativity, is natural.

Definition I. If a spatial point has a smooth world line $W \subset M$ and $\mathbf{k}$ is a vector tangent to W at an event $\lambda \in W$ and $f \in \mathbb{F}_\lambda$, then that spatial point is at rest in f at the event λ if and only if the time axis of f is parallel to vector $\mathbf{k}$.

Definition I has been given the form of a partial definition since for our needs it suffices to define the concept of rest in the quasiflat space-time U_λ. The assumption that W is smooth implies that for every $\lambda \in W$ the tangent vector $\mathbf{k}$ exists and is a nonzero vector.

The concept of rest[22] is related to the concepts of rigidity and distance by the theorem that two timelike straight lines are parallel if and only if they are rigidly connected. The rigidity is defined as a constant distance while the distance is naturally and *invariantly* defined in mathematical terms of the theory of relativity. [The definitions of distance and rigidity and the mentioned theorem are well formulated with respect to both curved (locally, i.e., in U_λ) and flat (globally) space-times in Synge (1964) and Synge (1965, Chapter I, Sections 16, 17, 20, and Chapter II, Section 4).]

4.3. Rods, Clocks, and Operational Definitions. Without entering into details we can say that the formalized physical theory is a mathematical theory in which some symbols relate to concepts from the empirical world and some expressions describe certain relations in that world. The procedure of assuming that certain symbols are the names of concepts from the empirical world and that some expressions are descriptions of that world is called physical interpretation of the given mathematical theory. For the sake of precision and uniqueness contemporary physical interpretation is realized in the spirit of operationism.

Since the Riemann space theory constitutes the mathematical basis of relativity, the geometrical character of the latter is determined. At the most basic, i.e., kinematic, level the physical interpretation of the theory of relativity is realized with the help of such concepts from the empirical world as rod (rigid meter stick) and clock. In operational terms we can say that space (distance) and time are such entities which are directly measured with a rod and a clock, respectively. (We are speaking here of direct measurements and not of indirect ones as, e.g., measurement of distance with a telemeter or with the help of a clock and motion, e.g., of a light signal.)

[22] We have become used to considering the concept of proper time (in the operational sense) as secondary with respect to the concept of rest, defining the former by the latter, but of course this convention may be reversed.

"It is neither the point in space, nor the instant in time, at which something happens that has physical reality, but only the event itself" (Einstein, 1956). However, in our elementary feeling the rod differs essentially from the clock. Thus, to be able to introduce a physical (or, more precisely, kinematic) interpretation we have to use a method that would allow to distinguish in the Riemannian geometry formalism between what is measured directly with the rod from what is measured directly with the clock. The method used today consists in distinguishing the signs of the appropriate invariants in the Riemannian space with an indefinite signature when only real coordinates are used. Once another method was sometimes used. It consisted in using together real and imaginary coordinates in the Riemannian space with a definite signature. The physical sense of the corresponding expressions in both languages is identical since those formalisms are isomorphic. The use of such formalisms makes it possible to introduce certain fundamental conventions at the mathematical level of physical interpretation of relativity, what is considered further in the present subsection.

Generally speaking, it is quite immaterial whether in the mathematical description of nature we use real or imaginary numbers and whether we do that everywhere and always or only in some instances. In many domains of physics complex numbers are simply used. One can even proceed much further and provide all or some scales of instruments (inclusive of rods and clocks) with imaginary or even complex numbers (the latter usually with proper constraints; see Appendix A). Such measures would not affect the physical essence of the description, so the relations between the results of measurements or their generalizations would remain the same. It is only the mathematical language that changes. That is why the discussion conducted between the supporters of the superluminal frame and transformation concepts on the subject whether the use of real, imaginary, or complex quantities [or even biquaternions (Imaeda, 1979)] is proper or not is completely futile. Truly essential is only the clear determination of the kind of instrument in the following relation: instrument – result of measurement – mathematical notation. If we assume such a standpoint, we liberate ourselves from various viewpoints presented by supporters of the superluminal frame concept and therefore we are able to assume an attitude toward them all.

In accordance with the attitude presented above, we shall introduce the following operational definitions:

Definition II. A vector $\mathbf{k} \in \mathbb{V}_\lambda$ is a spacelike (timelike) vector if and only if there exists $f \in \mathbb{F}_\lambda$ such that the direct measurement in M at the event λ in the direction of $\mathbf{k}$ is realizable in f by means of a rod (clock) alone.

Definition III. A vector $\mathbf{k} \in \mathbb{V}_\lambda$ is a null vector if and only if for every $f \in \mathbb{F}_\lambda$ the direct measurement in M at the event λ in the direction of $\mathbf{k}$ is not realizable in f by means of a rod alone and it is not realizable in f by means of a clock alone.[23]

Note that in Definition III the null vector is defined operationally without using the term *velocity of light* and that Definitions II and III imply the following: for every $\mathbf{k} \in \mathbb{V}_\lambda$, vector $\mathbf{k}$ is not null if and only if it is spacelike or timelike (equivalently: $\mathbf{k}$ is null if and only if it is neither spacelike nor timelike). It means that every $\mathbf{k} \in \mathbb{V}_\lambda$ is null or spacelike or a timelike vector in the operational sense of Definitions II and III.

Also note that it does not follow from Definition II that the spacelike (timelike) vector cannot be simultaneously timelike (spacelike) and that Definitions II and III have not been preceded with a proof or assumption stating that the defined vectors exist. The fact that the spacelike vector cannot be timelike and vice versa (what could have been provided for in Definition II by a suitable limitation of the definiens range but was unnecessary as we shall see in a moment) as well as the fact that the defined vectors exist result from the physical interpretation of Riemannian spaces. Eventually, the theory of relativity requires such Riemannian spaces for which the methods of distinguishing between space and time, of determining the photon world lines, etc. may be applied. For those Riemannian spaces it is proved that such and only such vectors exist in $\mathbb{V}_\lambda$'s which can further be physically interpreted as spacelike, timelike, or null vectors. That interpretation excludes conjunction of those features for one vector, for instance, if the interpretation consists in distinguishing the signs of invariants or in separating the features of the vectors by means of the light cone (for the latter see Section 4.4). This level of physical interpretation can be called mathematical, whereas the level where the concepts of rod and clock are used can be called operational. At the mathematical level of interpretation certain fundamental conventions are introduced that characterize the given theory as a physical one. Those conventions are determined on the one side by our opinions on the given part of reality of nature and on the other side by the abilities of the language of the given mathematical theory used for describing that part of nature. For instance, in the theory of relativity

[23] Here we have an implicit assumption that luminal f does not exist in the operational sense, which is in fact in accordance with the operational interpretation of the Lorentz transformation. The concept of luminal f has been introduced in Lord and Shankara (1977, Section 5); however, apart from the attractive name ("photon rest frame") nothing is to be found there. A standard transformation has been made in the quoted paper from two Lorentz coordinates to two null coordinates (what already collides with the definition of f, since those null coordinates are nonorthogonal, cf. Section 4.2) and it has been called "Lorentz transformation with velocity $v = c$" forgetting probably that the Lorentz transformation is singular for $v = c$.

among such conventions we have, e.g., the convention stating that the given vector may not have two of the features of being spacelike, timelike, or null, or the convention stating that the number of timelike dimensions and of spacelike dimensions of M are invariants of transformations. (We do not analyze here the mutual relations between such conventions nor between the concepts of the theory of relativity.) Negation or modification of even one of such conventions is equivalent to rejecting or modifying, respectively, the given theory as a physical theory. For instance, one of the forms of modification consists in making extensions of the given theory so that the given convention holds in the old nonextended part of a new extended theory but does not hold in the whole of the new theory. We shall encounter such situations in Section 4.4.

If we accept the standard definition of f (see Section 4.2) and the standard physical interpretation of relativity as a whole, i.e., at both the mathematical and operational levels, then we can easily prove that Definitions II and III are equivalent to the standard definitions of spacelike, timelike, and null vectors expressed in terms of the Riemannian geometry.

Definition I is a standard definition since it does not contain operational terms. If, however, we accept Definition II, then Definition I becomes operational since the operational term appears implicitly in the expression "time axis."

When speaking in the further text of the spacelike, timelike, and null vector or direction we shall understand those expressions in terms of Definitions II and III.

4.4. The Superluminal Frame and the Theory of Relativity. The present section involves the important assumption that M is an m-dimensional space-time having everywhere *only one* timelike dimension, what—as a matter of fact—is the basic assumption of the theory of relativity (for $4 \geqslant m \geqslant 2$; comments on the space-times with a greater number of timelike dimensions can be found in Section 4.6). That space-time will be called the *usual* space-time. For such a space-time we can easily define the light cone as a two-sheet $(m-1)$-dimensional region $\mathbb{N}_\lambda \subset \mathbb{V}_\lambda$ whose vertex is fixed at an arbitrarily chosen event $\lambda \in M$. Let the m-dimensional regions $\mathbb{S}_\lambda, \mathbb{T}_\lambda \subset \mathbb{V}_\lambda$ be the exterior and interior of the light cone $\mathbb{N}_\lambda$, respectively, i.e., $\mathbb{S}_\lambda \cup \mathbb{T}_\lambda \cup \mathbb{N}_\lambda = \mathbb{V}_\lambda$ and $\mathbb{S}_\lambda \cap \mathbb{T}_\lambda = \mathbb{T}_\lambda \cap \mathbb{N}_\lambda = \mathbb{N}_\lambda \cap \mathbb{S}_\lambda = \mathbb{O}$. The elements belonging to $\mathbb{S}_\lambda$, $\mathbb{T}_\lambda$, and $\mathbb{N}_\lambda$ and only the elements belonging to those sets are denoted by the symbols **s**, **t**, and **n**, respectively (the synonyms are $\mathbf{s} \in \mathbb{S}_\lambda, \mathbf{t} \in \mathbb{T}_\lambda, \mathbf{n} \in \mathbb{N}_\lambda$). By definition, $\mathbb{N}_\lambda$ separates all vectors **s** from all vectors **t** in $\mathbb{V}_\lambda$. Let us observe that that separation is a purely geometrical fact as long as we do not apply physical interpretation. The use of that interpretation (as a whole, see Section 4.3) means that we assign the vectors **s**, **t**, and **n** the features of being spacelike, timelike, and null, and only such features, respectively. Then from Definition I it follows directly that in the case of usual space-time no such f exists in which the tachyon would be at rest (more precisely, no such $f \in \mathbb{F}_\lambda$ exists in which the tachyon is at rest at λ for every $\lambda \in M$). In other words, for the usual M we have not the concept of rest (immobility) in relation to the tachyon, and thus the concept of proper time of the tachyon (see footnote 22). Hence in the class of noncontradictory concepts related to the theory of relativity we have neither the concept of superluminal f, and nor that of superluminal transformation (cf. Section 4.2).

In that situation the fundamental question arises: is the concept of superluminal f contradictory to or independent of the mentioned class of concepts? If it is contradictory, then there is no extension of the theory of relativity which would include the concept of superluminal f and the problem is terminated. If, however, it is independent, then such an extension does exist.

Assume that this latter case occurs, i.e., we accept the whole class of noncontradictory concepts related to the theory of relativity and add to that class the independent concept of superluminal f. Let us denote the class extended in that way together with its consequences by *SFE* (science fiction extension). We shall denote the superluminal f by the symbol F and consider $\mathbb{V}_\lambda$ such that λ is the origin of F. By definition every F is f, hence by definition our $F \in \mathbb{F}_\lambda$. Thus by virtue of Definition III vectors **n** are null also in the sense of F (i.e., they are operationally verified as null vectors by the observer being at rest in F), and only the **n** vectors are null in the sense of F. Since $F \in \mathbb{F}_\lambda$, then by virtue of Definition I one of the vectors **s** must be the time axis of F (figuratively, the direction in M at λ of the proper time in F is the direction of that s), and hence that s is a timelike vector in the F sense. Note that this is not pure verbalism, but that the problem concerns a kind of measuring instrument (Definition II). Thus the assumption of *SFE* implies a modification of the fundamental convention stating that the given vector has not the feature of being simultaneously spacelike and timelike (see Section 4.3). Note also that Definitions I, II, and III are formulated so that they operate in *SFE* with no need of modification.

If one would like to postulate that some other vector **s** is spacelike in the F sense, then one would have to assume a discontinuous physical interpretation over the continuous distribution of vectors **s** in $\mathbb{S}_\lambda$, since $\mathbf{n} \notin \mathbb{S}_\lambda$. The problem whether such an *SFE* is intrinsically contradictory or not is a matter of discussion in the domain of the methodology of physics. Note that there is no analogy here with the determination of a discontinuous function on a continuous set of its arguments but here discontinuity is in the physical interpretation. Assuming such a discontinuous interpretation we must accept the existence of, e.g., such a phenomenon in F that a rod at rest when slightly moved transforms into a clock, i.e., an instrument measuring time directly (and not via motion). More figuratively, a homogeneous piece of metal held in the hand would become during a slight motion a pocket watch with dial and machinery. These are the consequences when there is no null vector between the spacelike and timelike ones, and we know that $\mathbf{n} \notin \mathbb{S}_\lambda$. Postulating a discontinuous interpretation would certainly be a new quality in methodology of physics.

However, if somebody would renounce paying such a price and would require that *SFE* preserves the continuous physical interpretation, then he would have to assume that all **s**'s are timelike vectors in *F* sense and that all **t**'s are either timelike (first case) or spacelike (second case) vectors in the *F* sense, as $\mathbf{n} \notin \mathbb{T}_\lambda$ and $\mathbb{N}_\lambda$ separates in $\mathbb{V}_\lambda$ all **t**'s from all **s**'s.

The first case means that *SFE* requires modification of the fundamental convention which states that the number of timelike and the number of spacelike dimensions of *M* are transformation invariants (see Section 4.3), since we obtain m timelike dimensions (in the *F* sense; with the simultaneous presence of null directions). Besides, in this case it is not possible to determine uniquely the state of being at rest in *F* since we have m independent times (m time axes). If somebody thinks that only one state of rest may exist, then he must assume that $F \notin \mathbb{F}_\lambda$ (cf. Section 4.2) what is contradictory to the earlier conclusion that $F \in \mathbb{F}_\lambda$. But somebody else may want such a version of *SFE* in which it is assumed that there may be m different states of rest in *F*. The observer's world who is at rest in *F* is a world without spatial dimensions and only with different kinds of times, what refers to all phenomena, i.e., subluminal, luminal, and superluminal in relation to the observer. Many questions arise here, e.g., what are for that observer clocks, and in general what are for him physical objects; maybe they are only fields in different times, since **n**'s are null vectors in the *F* sense, but then what is velocity if there is no space, etc., etc.

In the second case we have a spatial dimension in the *F* sense but only one. Here the situations are different for the subcases $m = 2$ and $m > 2$, which is due to that the topological structures of regions $\mathbb{S}_\lambda$ and $\mathbb{T}_\lambda$ are the same if $m = 2$, whereas they are different if $m > 2$. For $m = 2$ $\mathbb{S}_\lambda$ converts to $\mathbb{T}_\lambda$ (and vice versa) as a result of rotation around λ, which is impossible for $m > 2$ (cf. Yaccarini, 1974).

In the subcase $m = 2$ the space-time remains a usual one and there is only one rest state (one time axis only) for the observer in *F* analogously as for the observer in subluminal *f*. The supporters of the idea of *F* have revealed a great deal of ingeniousness in this subcase when constructing their models of "physical" worlds, but the way to follow seems still long. Here also many embarrassing questions can be put. Furthermore the subcase $m = 2$ is unphysical.

In the subcase $m > 2$, like in the first case, we must modify the convention assuming that the numbers of kinds of dimensions of M are transformation invariants (see Section 4.3) and we must agree to the existence of $m - 1$ independent times in the F sense together with the corresponding implications. We can speak here of velocities ($m - 1$ different kinds) and simultaneously interpret the vectors $\mathbf{n}$, since we have one spatial dimension in the F sense, but the number of embarrassing problems is as large as in the first case.

We have presented above some situations "seen" by the observer being at rest in F in all possible versions of *SFE*. It should be strongly emphasized that these are the situations in *our* space-time M which by assumption is for us the usual space-time and for which we have permitted the existence of F. All the observers, both subluminal and superluminal (if the latter exist), find themselves in one common space-time by virtue of the definitions of concepts of transformation and reference frame (Sections 4.1 and 4.2).

We have presented above the consequences of adding the concept of F to the class of noncontradictory concepts related to the theory of relativity. Note that if any of the versions of *SFE* is formulated without logical contradictions, then that version will be sound from the methodological point of view (cf. Section 2.1) and its acceptation or rejection is only a matter of opinion.

Now we can emerge from the stream of science fiction but already with the following conclusion: the concept of such an F in whose sense the space-time is a usual one (i.e., the world seen by the observer being at rest in F is a normal world) cannot be consistently added to the class of noncontradictory concepts related to the theory of relativity when $m > 2$. Thus, the extension of the theory of relativity to a theory with such F does not exist for all $m > 2$ inclusive of the physical case $m = 4$. This is due to the fact that $\mathbf{n} \notin \mathbb{S}_\lambda$ for every $\mathbf{n}$.

Our considerations refer to an arbitrary event $\lambda \in M$, so the above conclusion is valid both for the special and general relativity cases. This means that if we accept the theory of relativity, and today we have no other choice (cf. Section 1), then we have to reject the possibility of three-spatial-dimensional rigid macroscopic objects in F existing. Thus we are obliged to abandon, among other things, the hope of our journey by a faster-than-light spaceship in a well-furnished cabin. Note that the following question: "can an observer being at rest in a subluminal f see the three-spatial-dimensional rigid macroscopic tachyon?" is wrongly formulated because the expression "rigid macroscopic tachyon" does not have an invariant operational sense in terms of the theory of relativity (cf. last paragraph of Section 4.2). This does not mean of course that we have to reject (from the point of view of the relativity) the possibility of faster-than-light phenomena existing, since firstly F's are not necessary for the observation of such phenomena and usual subluminal f's suffice, and secondly the tachyons may be microscopic objects or may be systems of suitably shaped fields expanding with the velocity of light, as for instance in the tachyon model presented in Kowalczyński (1979).

In general, one has the impression that special relativity is too tight for tachyonic problems, whereas in general relativity—as it is commonly known—bradyonic, luxonic, and tachyonic solutions have equal rights.

4.5. General Criticism of the Works Supporting the Superluminal Frame. The supporters of the concept of F rely on the theory of relativity (special) but they protest against their F's being such as those in *SFE*, and that is not to be wondered. They want the F's to be usual f's. As we already know from Section 4.4 that requirement is unfeasible being self-contradictory (surely for $m > 2$). Thus the embarrassing problem arises: how could it happen that an extensive literature exists about an empty concept? Two explanations can be given.

Let us pass for the sake of simplicity from U_λ to the whole usual Minkowski space M (cf. Section 4.2) and consider the physical case $m = 4$. We shall thus operate there where the supporters of the idea of F operate (the unphysical case $m = 2$ is excluded here). Assume a system of Lorentz coordinates x, y, z, t in M. For better visuality the following standard one-to-one mapping is made:

$$\phi_\lambda : \mathbb{V}_\lambda \to M \tag{3}$$

where $\lambda \in M$ and $\phi_\lambda : \mathbb{S}_\lambda, \mathbb{T}_\lambda, \mathbb{N}_\lambda \to S, T, N$, respectively, and where $S, T, N \subset M$. The two-sheet three-dimensional region N is commonly (though unprecisely) called the light cone in M with vertex at the origin of the coordinate system. Then the four-dimensional regions S and T are the exterior and interior of that light cone, respectively. Let the two-sheet three-dimensional region $P \subset M$ be a cone with axis x and tangent to N. Let the four-dimensional region Q be the interior of P. Of course we then have $Q \subset S$. Let M' be the second usual Minkowski space with the corresponding S', T', N', P', and Q' regions and provided with a system of Lorentz coordinates x', y', z', t' (see Figure 2).

The first explanation of the raised problem is as follows. The supporters of the ideas of superluminal frame and transformation have used the terms *frame* and *transformation* illegally since in reality they make a one-to-one mapping ϕ_1 of M onto M' (see Section 4.1):

$$\phi_1 : M \to M' \tag{4}$$

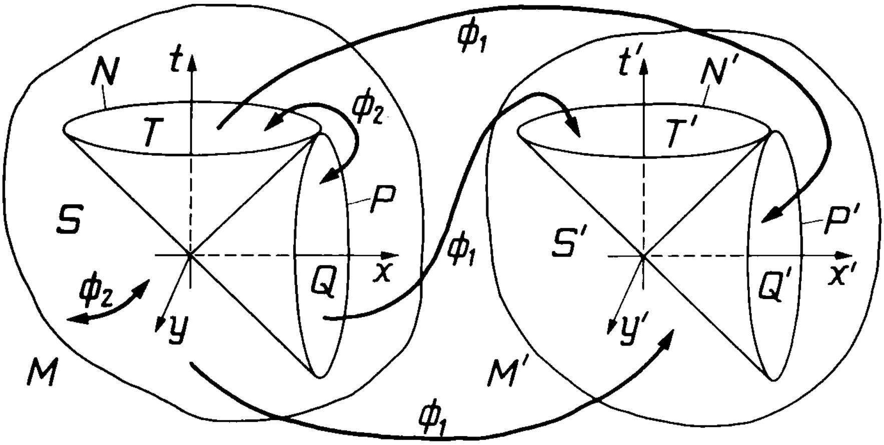

Fig. 2. The mappings ϕ_1 and ϕ_2, erroneously called superluminal Lorentz transformations. The picture relates to the case $m = 3$. Explanations in the text. The second sheets of N, P, N', and P' cones are not shown. After the section $y = 0$ is made we get the picture of the situation in the unphysical case $m = 2$ where $N = P$ and $S = Q$.

More precisely this mapping looks as follows (see Figure 2):

$$\phi_1 : N \to P', \qquad \phi_1 : P \to N', \qquad \phi_1 : T \to Q',$$
$$\phi_1 : Q \to T', \qquad \phi_1 : S - P - Q \to S' - P' - Q' \tag{5}$$

Of course such a mapping can also be understood as mapping ϕ_2 of M onto itself:

$$\phi_2 : M \to M \tag{6}$$

if we assume that $M = M'$. A more precise presentation of ϕ_2 is given by relations (5) if we abandon there the primes (see Figure 2). The mappings ϕ_1 or ϕ_2 are given in many papers as explicit formulas [i.e., as an explicit form of equations (1)] under an unfortunate name of "superluminal Lorentz transformations." With the use of those formulas one can easily verify relations (5). The symbol v (or u) occurring in those formulas, which is given there the meaning of superluminal velocity, obviously has nothing to do with velocity, since it has no sense to speak of velocity of space-time (as a whole or its part in relation to another part). The number v occurring there is a parameter of mapping (1). (Formally we have there four classes of mappings ϕ_1 or ϕ_2 determined by combinations of signs of the parameter and root.) The fact that in the criticized works the mappings really appear is the source of trouble with which the authors struggle. Possibly, such things as the statement that superluminal transformation changes the sign of the invariant (see Appendix B), the concept of tachyon corridor (Marchildon et al., 1979) or the sentence "In conclusion we have obtained as a result that tachyons and bradyons live in two *different* metric spaces" (Goldoni, 1973, p. 515) are the results of those struggles.

The second explanation of the problem raised at the beginning of the present section is quite simple. The supporters we are considering really made transformations from frame to frame, but those were usual subluminal Lorentz transformations and f's (Yaccarini, 1974). Simply, if we assume as they did that in their transformation formulas the symbols x and t denote the spacelike and timelike coordinate, respectively, then really (in the operational meaning) x' is a timelike and t' is a spacelike coordinate in those formulas, i.e., quite opposite to what they have assumed (the problem whether the y, z, y', and z' coordinates are real or imaginary is immaterial; see Appendix B). Thus the symbol v (or u) appearing in those formulas does not represent velocity but its inverse (for $c = 1$; cf. Gilson, 1968; Mariwalla, 1969; Naranan, 1972; Yaccarini, 1974).

As regards the Goldoni idea (Goldoni 1973, 1978) each of the two above explanations should be applied three times. For the first time as shown above and for the second and third times by substituting y, y' and z, z' for x, x', respectively.

4.6. Comments on the Works on Time Multidimensionality. The idea of multidimensional time (for references see Ziino, 1980, and Pavšič, 1981) has also been harnessed to the problem of superluminal frame. Usually authors refer to three-dimensional time, and in consequence to six(three-complex)-dimensional space-time, though there has been a work considering the problem in the twelve(six-complex)-dimensional space-time (Ramon and Rauscher, 1980) (see last paragraph in Appendix A). Those times and the independent times that would be detected by the observer (if he did exist) at rest in F and mentioned in Section 4.4 should not be confounded. In the case considered now the independent times have to exist, by definition, also for the subluminal observer.

From the standpoint of operationism we can ask the supporters of the multidimensional time idea where have they seen clocks measuring those different times (cf. 2nd paragraph in Section 4.3) of course apart from clocks out of order. Though Pavšič (1981) seems to suggest that those additional times of ours would be detectable only by a superluminal observer [relations (2), (3), and (2') in Pavšič, 1981], he gives no clear explanation. In turn, the additional times of the superluminal observer would be detected by us, though it is not known how (these are my guesses, since Pavšič does not say that clearly). This is not important, however, since condition (ii) and the remarks as to the change of quadratic form sign in Pavšič (1981) mean, in accordance with what has been said here in Section 4.1 and Appendix B, that in Pavšič (1981) we have in fact a mapping between two six-dimensional flat space-times which have four-dimensional common parts. Hence, there is neither a superluminal transformation nor a superluminal frame and the problem collapses.

The embarrassing question regarding the clocks is probably not unfamiliar to the supporters of the multidimensional time idea, and that is probably why it is generally stated by them that the clocks measure only the quantity τ where[24]

$$|\tau| = \left(t_x^2 + t_y^2 + t_z^2\right)^{1/2} \tag{7}$$

but then what is the physical meaning of the quantities t_x, t_y, and t_z and what is their use (cf. Lee and Kalotas, 1977, p. 370). Perhaps t_x, t_y, and t_z are meant to have the sense of mathematical auxiliary quantities intended to describe six-dimensional transformations between the observables x, y, z, τ and x', y', z', τ', which seems to be indirectly suggested by some authors (see, e.g., Mignani and Recami, 1976) and which in fact collides with the spirit of relativity by breaking the equivalence of the logical types of those observables, but then the question arises as to the mutual uniqueness of those transformations. In fact there is an infinite number of quantities t_x, t_y, and t_z that fulfil equation (7) for a given value $|\tau|$. What would then such transformations determine?

Ziino (1979a, b) tried to prove that our time has three independent components. His reasoning based on the light speed invariance has been criticized (Ray, 1979; Spinelli, 1979) and his defence (Ziino, 1980) was quite unconvincing. Ray (1979) rightly observed that light speed invariance is in Ziino (1979a, b) an irrelevant argument. We can add here the following obvious remarks against Ziino's argumentation. In kinematics the number of degrees of freedom determines the liberty (possibility) in describing, e.g., a world line. On the other hand a given pointlike object always has its world line determined by a system of equations containing only one independent parameter, and that is the case both in relativistic and nonrelativistic kinematics. It is a relative and insignificant matter whether that object is in motion or not.

[24]The term *three-vector* commonly used in these situations for the quantity τ (where it is denoted by the symbol $\mathbf{t}$) is unsuitable and deceiving since we have there six-dimensional M and $\mathbf{V}_\lambda$ ($\lambda \in M$).

It seems that the Ockham principle is worth reminding from time to time.

5. CONCLUSIONS

The existence of faster-than-light phenomena is not contradictory to the theory of relativity. The known tachyonic causal paradoxes do not occur if suitable physical conditions limiting the tachyonic phenomena exist. Whether it is possible to deduce such conditions inside the theory of relativity or not remains an open problem. Nevertheless such extensions of relativity surely exist where those conditions occur. The extension of the theory of relativity by adding the noncontradictory assumption that there exists an inertial reference frame preferred as regards tachyons but not preferred as regards bradyons, luxons, and all systems that are considered to be tachyonless can serve as an example.

The concept of superluminal reference frame does not exist in the theory of relativity. The extension of relativity by that concept implies conclusions which are strange from the physical point of view. On the other hand the concept of superluminal frame such as is used by its supporters, i.e., the concept of such a frame where the observer being at rest would see the world as a usual space-time, is contradictory to the theory of relativity and therefore cannot be added to that theory.

NOTE ADDED IN PROOF

Some problems discussed in Section 4 are presented less precisely but in a simpler and shorter way in my paper that will appear in *Acta Physica Polonica B 15* (1984), probably in No. 1 or 2.

APPENDIX A. IMAGINARY AND COMPLEX COORDINATES

In the case when the coordinates x^μ and $x^{\mu\prime}$ occurring in equations (1) are of one kind, i.e., they are either only real, or only imaginary or only complex, there are no problems with dimensions of the manifolds M and M', since in the two first cases M and M' are m-dimensional and in the third case they are $2m$-dimensional in the terms of real or/and imaginary dimensions. If we introduce the concept of complex dimension, then in the third case M and M' are of course m-dimensional.

In the case of mixture of those kinds of coordinates the question about the number of dimensions of M and M' arises, which is of course connected with the problem of the number of independent quantities. The simplest and most common convention is in that case the use of real or/and imaginary dimensions. In practice this reduces to the use of real dimensions only, since if there are any imaginary ones, then we can convert them by an additional simple transformation [see, e.g., equation (B3) in Appendix B] into real dimensions. For instance, if for a given $\lambda \in M$ there exists a base of m directions such that every direction is real or imaginary and at the same time complex coordinates occur in equations (1), then those complex coordinates must be involved in additional constraints so that the number of independent quantities be equal to m. For instance, for two independent complex coordinates x^1 and x^2 the assumption $x^1 = \overline{x^2}$ is often such a constraint. Such a situation imposes proper constraints also on the functions Y^μ, which results from the fact of relations (1) being equations.

In spite of the suggestions of some authors it should be emphasized that the problem of imaginary and complex quantities occurring both in the present work and in other works dealing with superluminal frames and transformations or multidimensional time does not relate to the so-called complex method in general relativity. That method, introduced into general relativity by Newman (1973) and subsequently intensely developed by many authors (for relevant references see Plebański, 1977), is a purely formal game. In that method no physical interpretations are made of the four-complex(eight-real)-dimensional manifolds and solutions of the complex Einstein equations. It is only after a real slice (Newman, 1973) is made [Rózga's prescription (Rózga, 1977)] and the usual four-real-dimensional space-time is obtained that a physical interpretation is applied.

APPENDIX B. THE SIGNS OF INVARIANTS

In many papers dealing with superluminal frames one encounters incomprehensible, at least for me, sentences saying that the superluminal transformation changes the sign of the "vector magnitude" (e.g., Recami and Mignani, 1974, p. 214). The authors of those papers operate by their own assumptions with the pseudo-Euclidean space, which is a special case of the Riemannian space. Let us make some very elementary comments.

At every nonsingular point of the Riemannian space the value of every invariant is independent of the choice of the coordinate system, so no transformation changes that value and thus the sign of the invariant (do not confuse pseudoinvariants). An example of invariant is the scalar product of **k** and **l** vectors, i.e., the quantity $k_\mu l^\mu$ $(= k^\mu l_\mu)$, which is called vector squared length or vector magnitude if $\mathbf{k} = \mathbf{l}$. Eventually, by virtue of the transformational definition of the vector we have

$$k_\mu = k'_\nu \frac{\partial x^{\nu\prime}}{\partial x^\mu}, \qquad l^\mu = l^{\nu\prime} \frac{\partial x^\mu}{\partial x^{\nu\prime}} \tag{B1}$$

hence,

$$k_\mu l^\mu = k'_\nu l^{\sigma'} \frac{\partial x^{\nu'}}{\partial x^\mu} \frac{\partial x^\mu}{\partial x^{\sigma'}} = k'_\nu l^{\sigma'} \delta^\nu_\sigma = k'_\mu l^{\mu'} \tag{B2}$$

Thus the equality $k_\mu l^\mu = k'_\mu l^{\mu'}$ is a mathematical fact and is independent of any physical interpretation of the quantities occurring in equations (B2). That fact results from the feature of the partial derivative of the composite function and from the algebraic feature of the product of determinants, in this case the Jacobi determinants of transformation (1) and its inversion. [Those Jacobi determinants exist and are nonsingular by virtue of the assumption that the system of equations (1) is a diffeomorphism if M is the Riemannian space (cf. Section 4.2).] Equations (B2) are fulfilled for every kind of x^μ and $x^{\mu'}$, i.e., both for real, imaginary, and complex coordinates, as well as for any mixture of those kinds of coordinates (cf. Appendix A). If therefore anybody maintains that a transformation (also the superluminal one if it existed) changes the sign of the quantity $k_\mu l^\mu$, then he falls into contradiction, since either there was no change of sign or no transformation.

It could be concluded from some papers (e.g., Recami and Mignani, 1974, pp. 216, 217) that the cause of the misunderstanding is the following. Let two systems of local coordinates x^μ and $x^{\mu'}$ be given for a certain $U \subset M$ such that $g_{11} \neq 0, x^1$ being real, $x^{1'}$ imaginary, and the first of equations (1) understood as a transformation being

$$x^{1'} = ix^1 \tag{B3}$$

while the remaining equations (1) being such that

$$\frac{\partial x^{\mu'}}{\partial x^1} = 0 \tag{B4}$$

for every $\mu \neq 1$. Then from equations (B3), (B4), and

$$g_{\mu\nu} = g'_{\rho\sigma} \frac{\partial x^{\rho'}}{\partial x^\mu} \frac{\partial x^{\sigma'}}{\partial x^\nu} \tag{B5}$$

we have

$$g'_{11} = -g_{11} \tag{B6}$$

(thus our transformation may change the signature; cf. 3rd paragraph in Section 4.3, and note the relative sense of the term *signature* when a mixture of real and imaginary coordinates is used in M). Let us consider vector $\mathbf{k}$ such that

$$k^\mu = \delta^\mu_1 k^1 \tag{B7}$$

and $k^1 \neq 0$. Its magnitude like that of every vector is an invariant of an arbitrary diffeomorphic transformation [equations (B2)], and in our systems of coordinates we have

$$k_\mu k^\mu = k_1 k^1 = g_{11}(k^1)^2 = k'_\mu k^{\mu\prime} = k'_1 k^{1\prime} = g'_{11}(k^{1\prime})^2 \tag{B8}$$

wherefrom by virtue of equation (B6) we get

$$(k^1)^2 = -(k^{1\prime})^2 \tag{B9}$$

which results also from equations (B1), (B3), (B4), and (B7). If incidentally $g_{11} = 1$, then $k_1 k^1 = (k^1)^2$. Some supporters of the idea of superluminal frame operated in the Lorentz coordinate system where $g_{\mu\nu} = \eta_{\mu\nu}$ and treated the coordinates y and z like our coordinate x^1 in equations (B3) and (B4). Thus, perhaps, they have confounded the scalar product $l_\mu l^\mu$ and the quantity $\Sigma_\mu (l^\mu)^2$ [cf., e.g., Ramon and Rauscher, 1980, equation (7), and Mignani and Recami, 1976, equations (3)], but that they should know best themselves. Anyway, the quantity $\Sigma_\mu (l^\mu)^2$ is not the squared length of vector **l**.

Another explanation of that misunderstanding is that they have confounded the transformation and mapping (Sections 4.1 and 4.5), since in the case of mapping equations (B1) and (B5) need not be valid. Incidentally, let us observe that the substitution of $x^{1\prime}$ for x^1 in an expression is not a transformation, e.g., after such a substitution equation (B3) would not be fulfilled. However, that substitution may be a mapping (cf. Section 4.1).

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